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## Artsy chaos: the secret life of a class of trigonometric sums

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**ARTSY CHAOS: The Secret Life of a Class of Trigonometric Sums**

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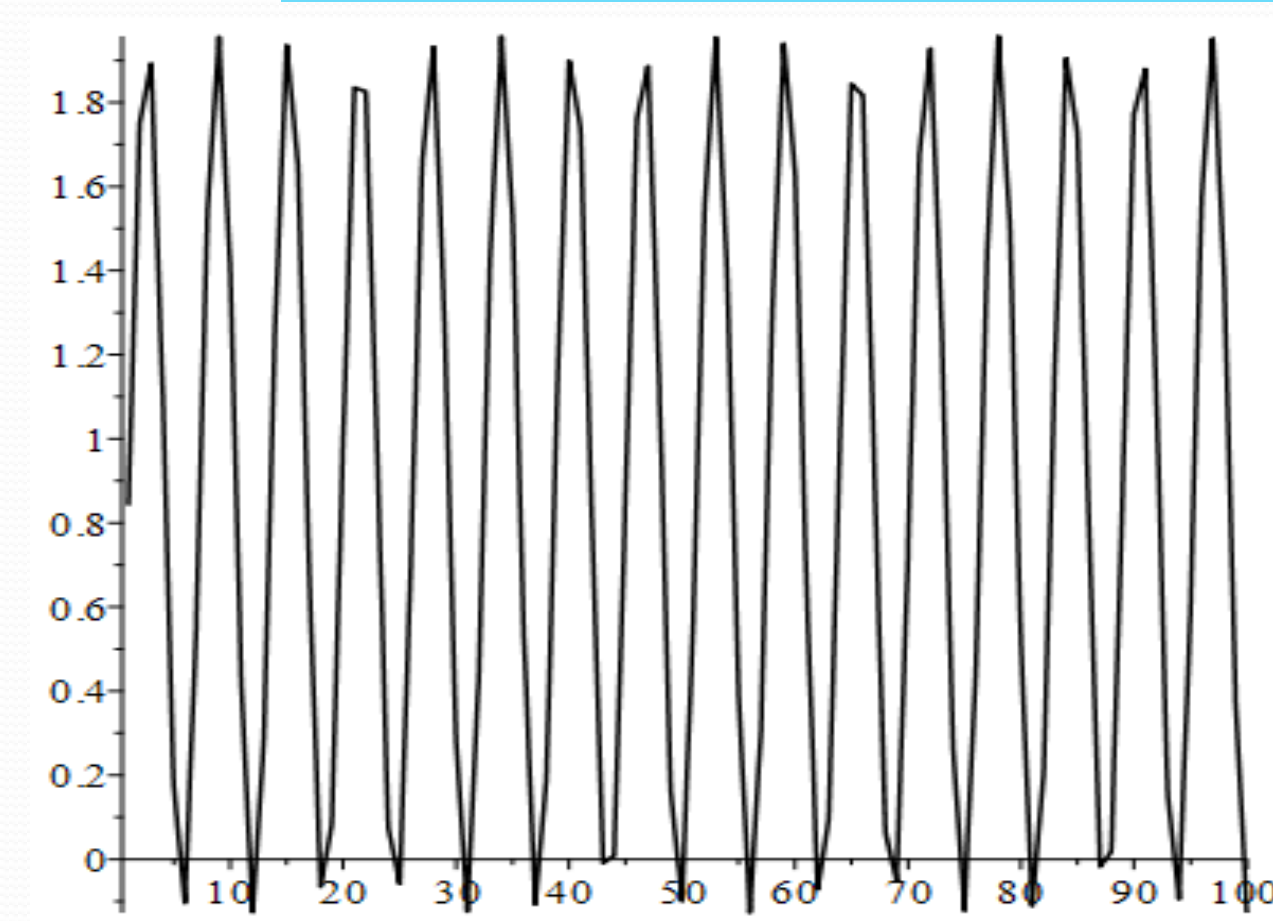
**ABSTRACT:** Classical trigonometric sums of the form  $\sum_{k=1}^n \cos k$  and  $\sum_{k=1}^n \sin k$  have simple closed form representations.

Our work considered changing the arguments of the trigonometric factors from  $\cos k$  (or  $\sin k$ ) to nonlinear analogues  $\cos k^\alpha$  (or  $\sin k^\alpha$ ), modulated with rotational terms of the form  $\omega^k$  for some complex  $\omega$  of modulus 1.

As an interesting outcome, for some exponents  $\alpha$  (such as  $\alpha \cong 1.29$ , a case that we studied in more detail) we discover surprisingly esthetic/"artsy" chaotic complex plots/regimes (courtesy of MAPLE)

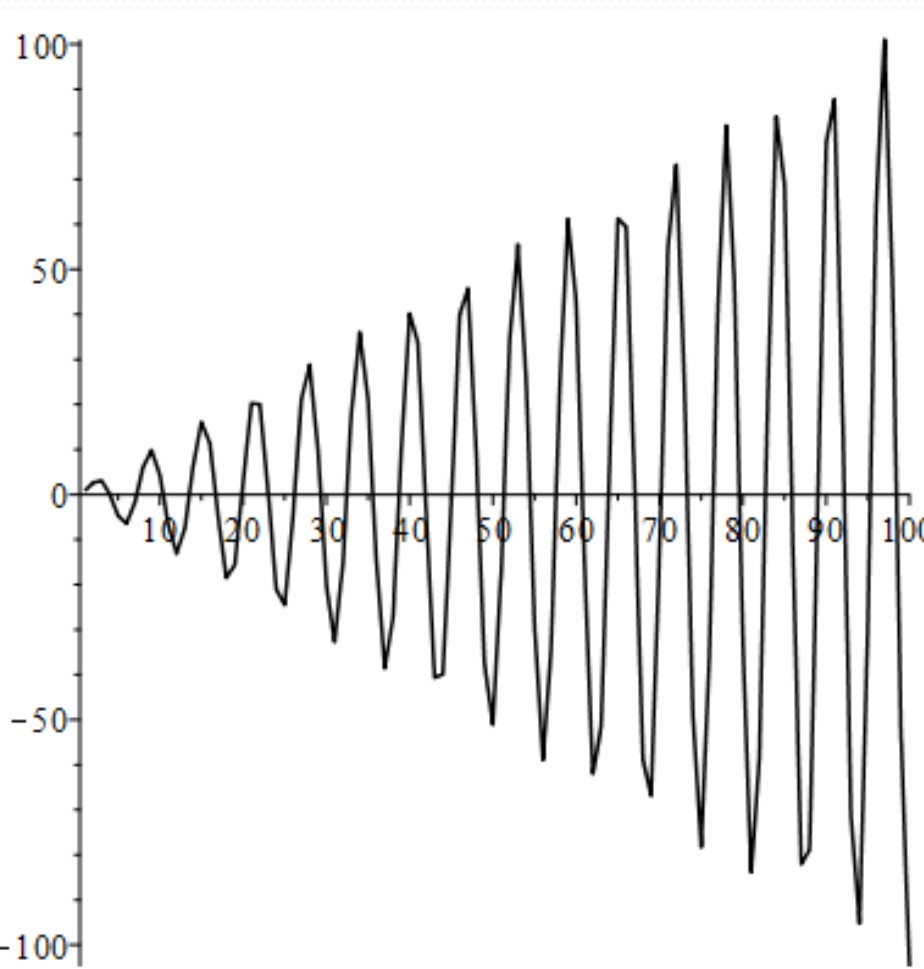
Classical trigonometric sums (straightforward, with  $\exp(ix) = \cos x + i \sin x$ ):

$$C_n := \sum_{k=0}^n \cos k = \frac{\sin\left(\frac{n+1}{2}\right)\cos\left(\frac{n}{2}\right)}{\sin\left(\frac{1}{2}\right)} \quad \text{and} \quad S_n := \sum_{k=0}^n \sin k = \frac{\sin\left(\frac{n+1}{2}\right)\sin\left(\frac{n}{2}\right)}{\sin\left(\frac{1}{2}\right)}$$

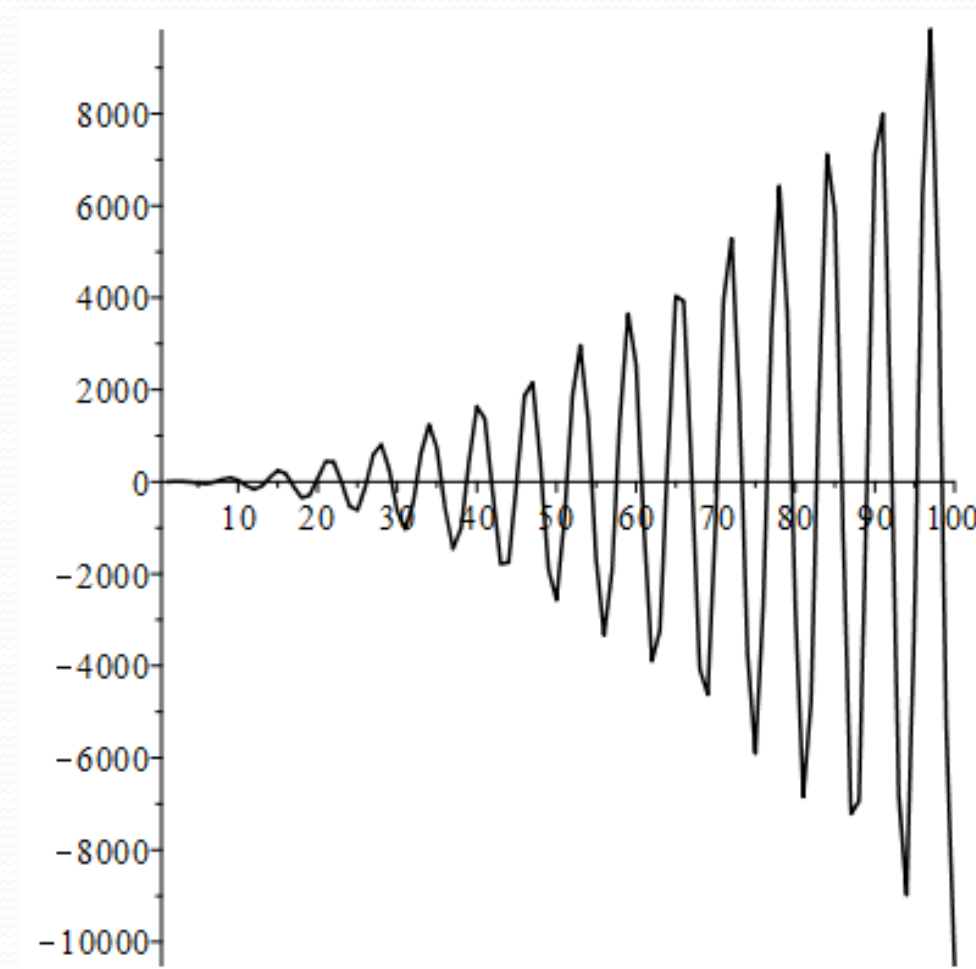


Ex.  $S_n = \frac{1}{2} \cot\left(\frac{1}{2}\right) \frac{\cos\left(n + \frac{1}{2}\right)}{2 \sin\left(\frac{1}{2}\right)}$

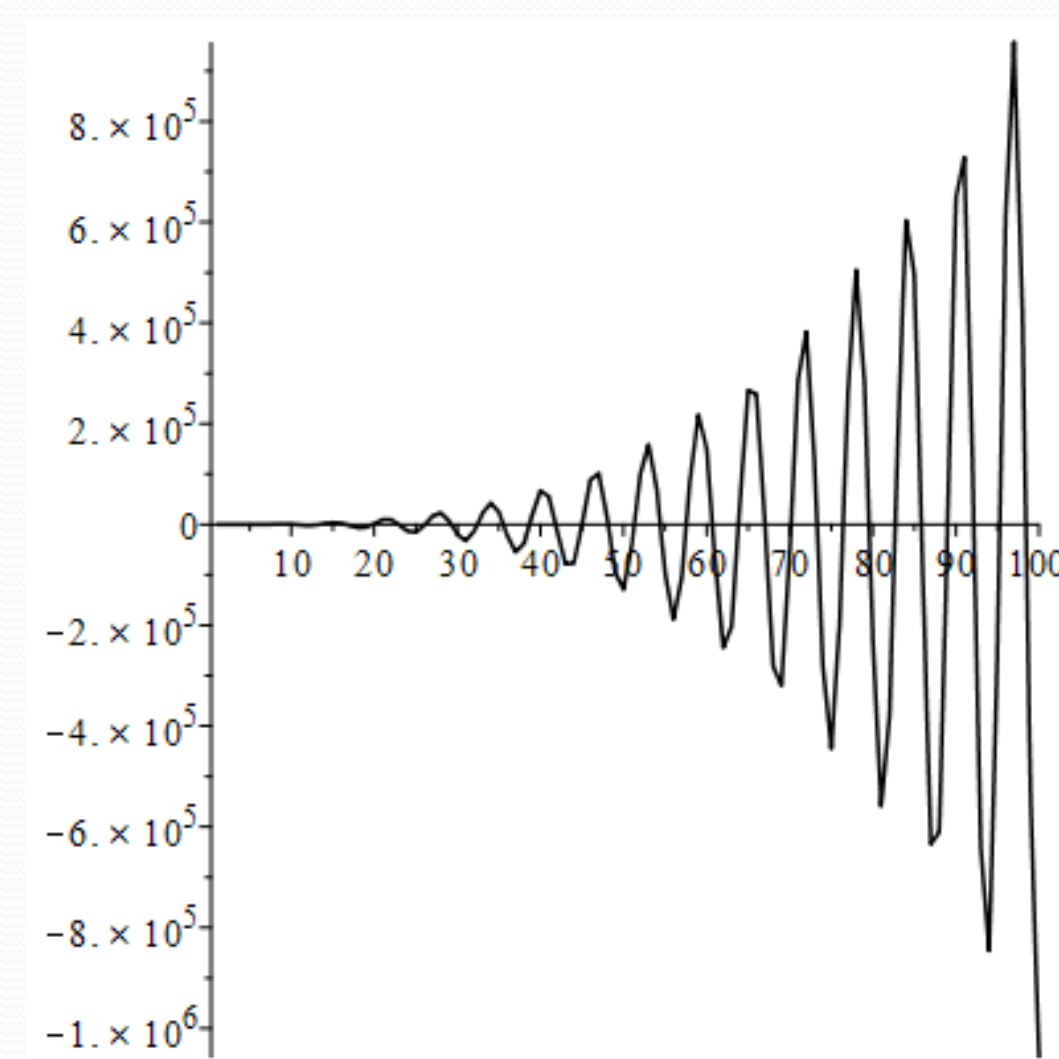
clear, controlled oscillatory range



$n \mapsto \sum_{k=0}^n k \sin k$

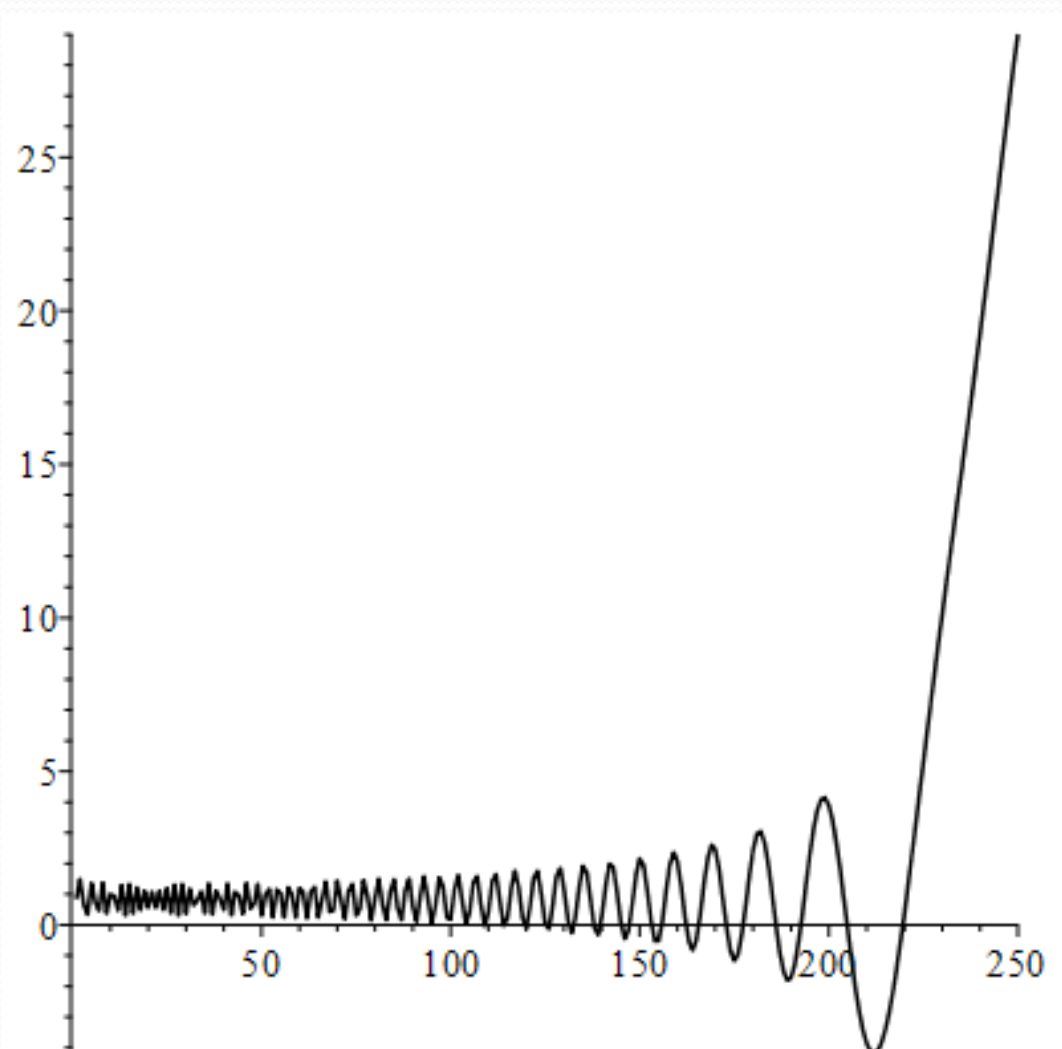


$n \mapsto \sum_{k=0}^n k^2 \sin k$

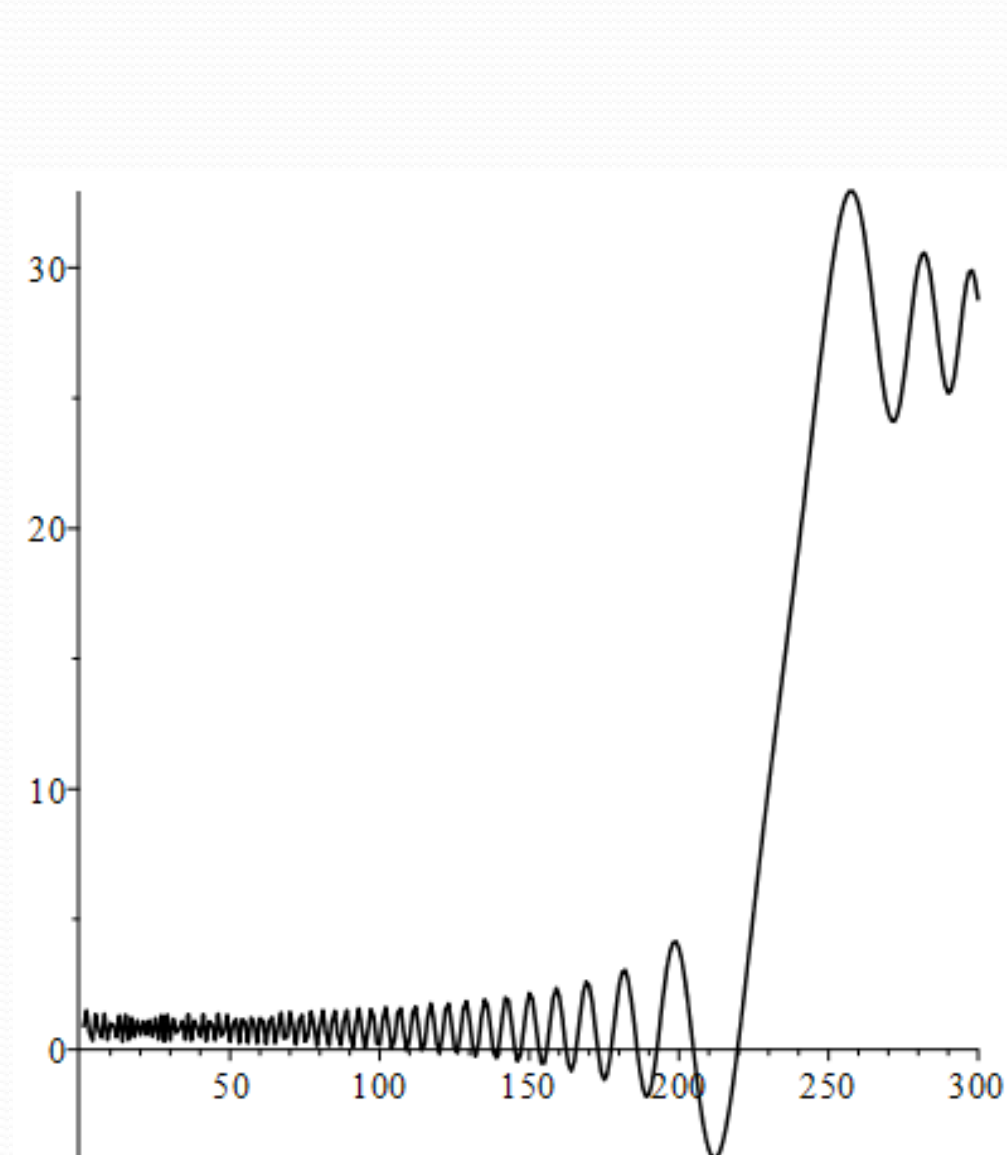


$n \mapsto \sum_{k=0}^n k^3 \sin k$

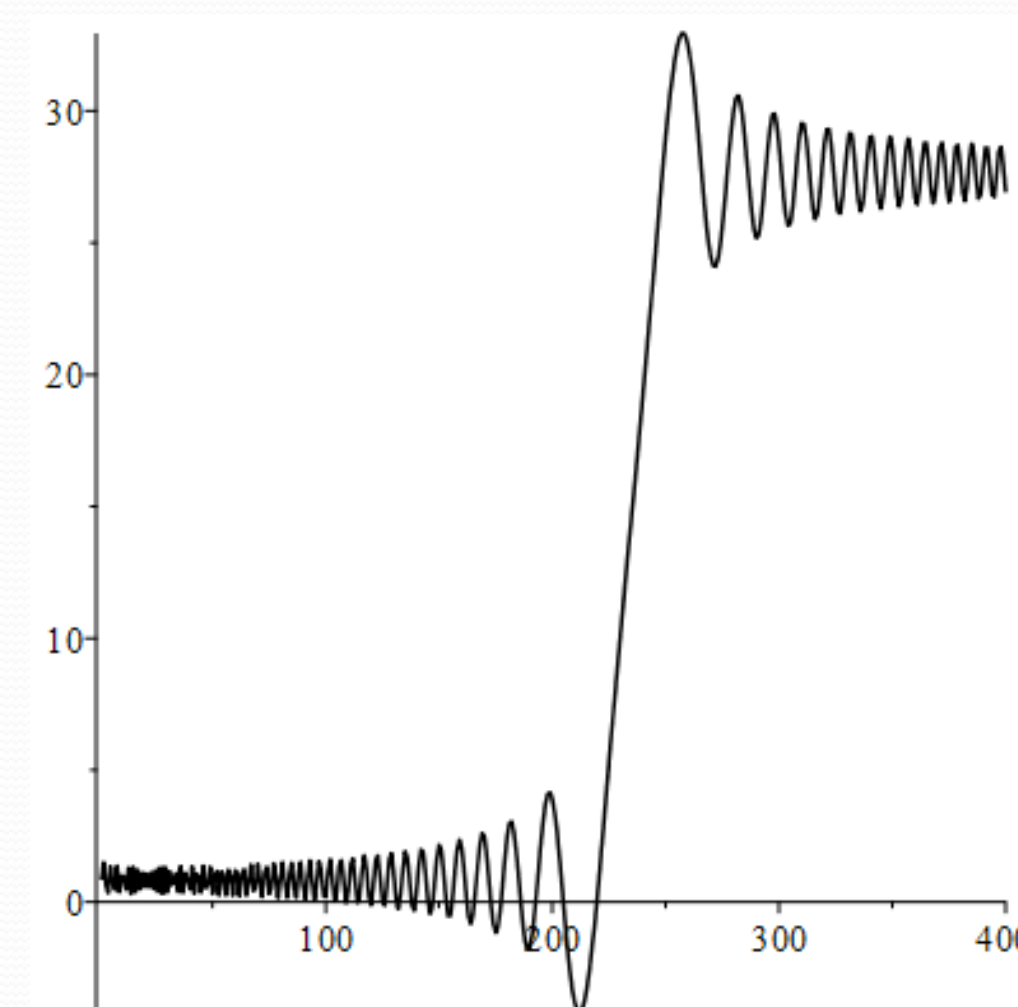
→ again, deterministic, expected



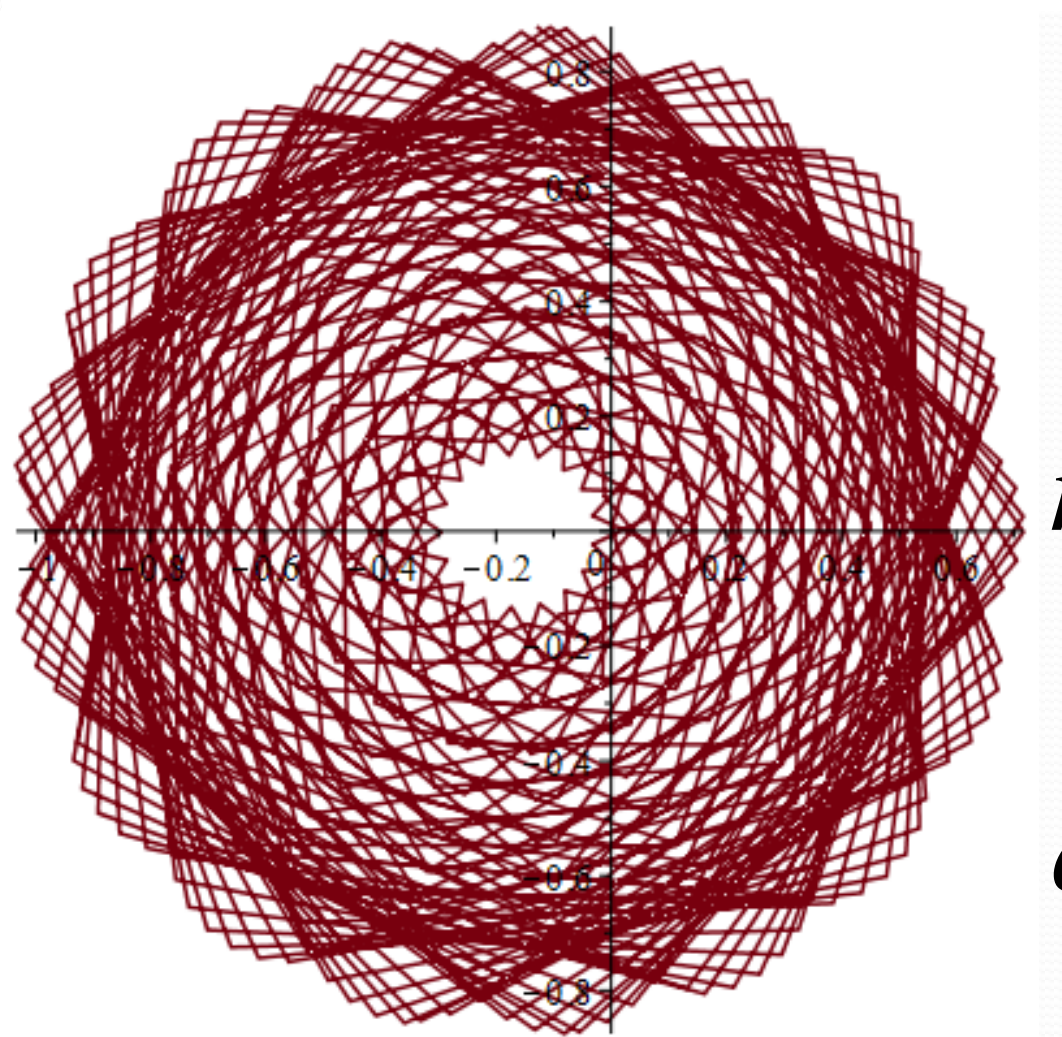
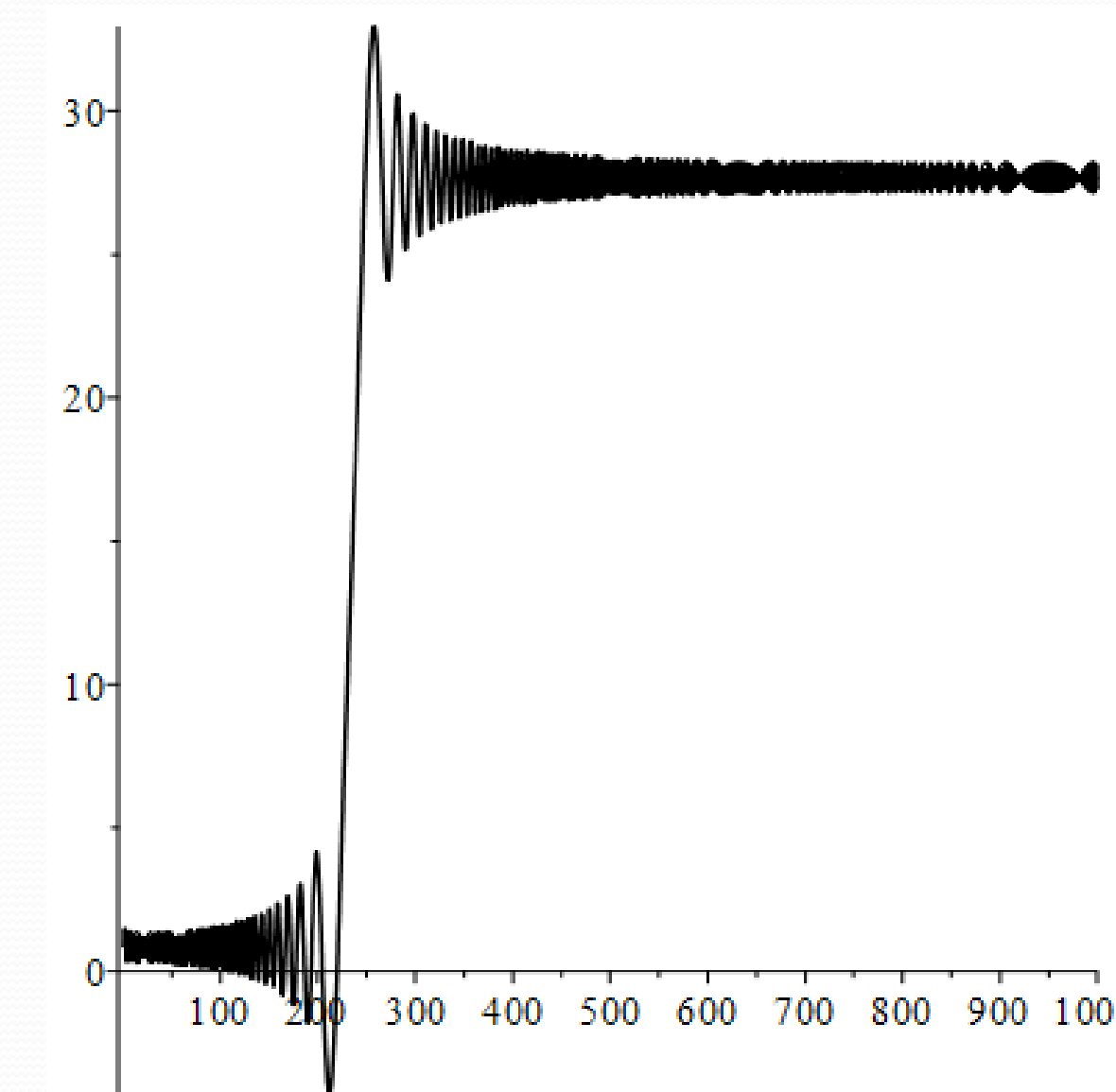
$n \mapsto \sum_{k=0}^n \sin(k^{1.29})$   
 $1 \leq n \leq 250$



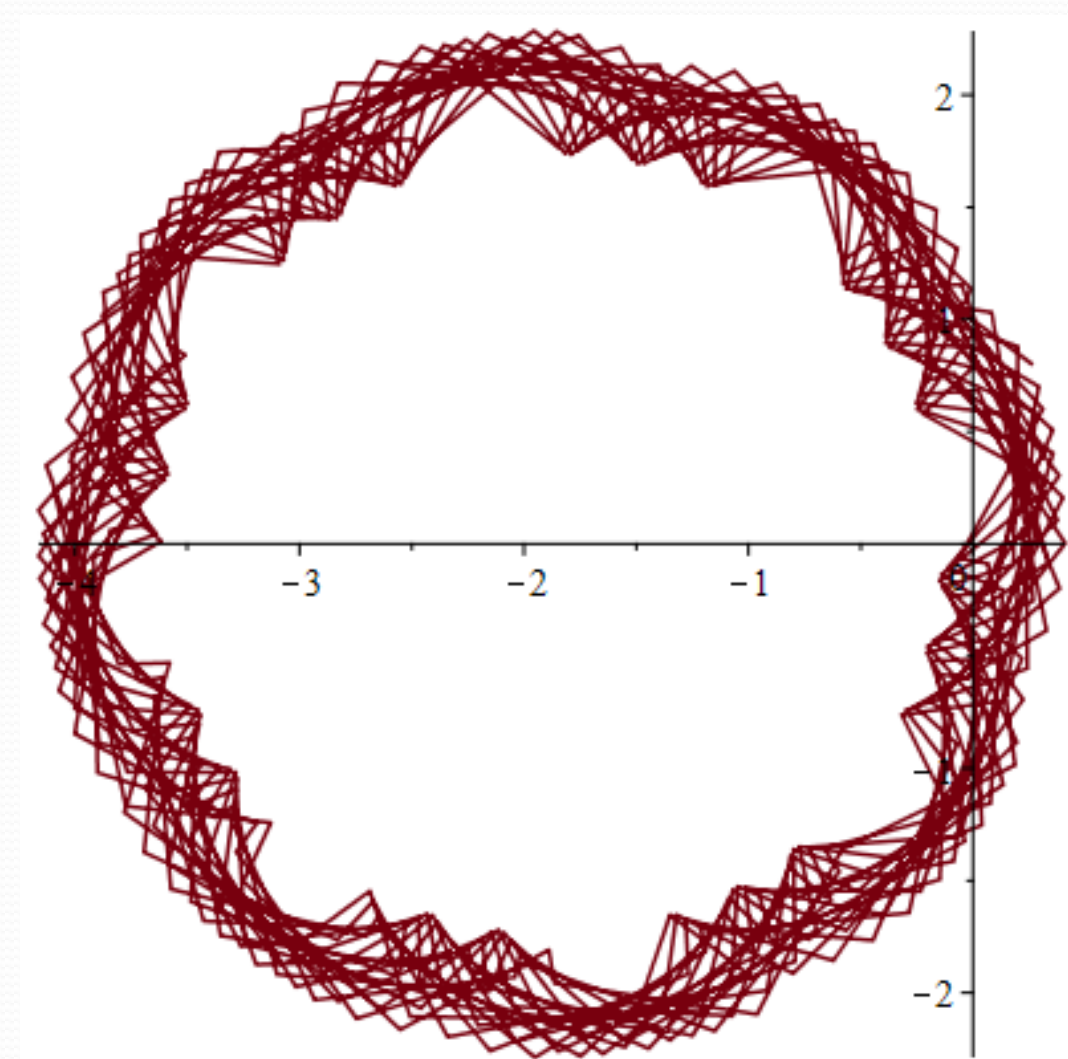
$n \mapsto \sum_{k=0}^n \sin(k^{1.29})$   
 $1 \leq n \leq 300$   
shifts oscillatory regime!  
regime!  
sort of unexpected



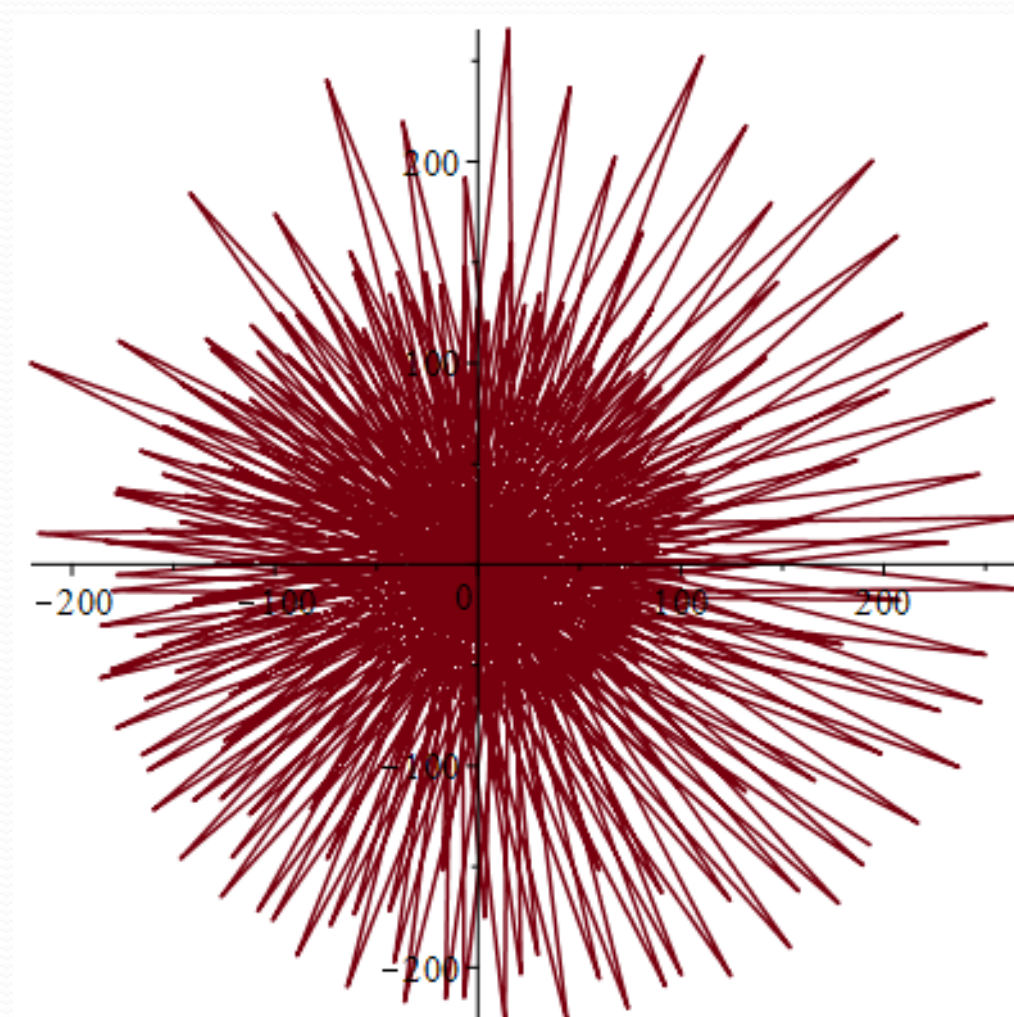
$n \mapsto \sum_{k=0}^n \sin(k^{1.29})$   
 $1 \leq n \leq 400$   
locally stable shift!



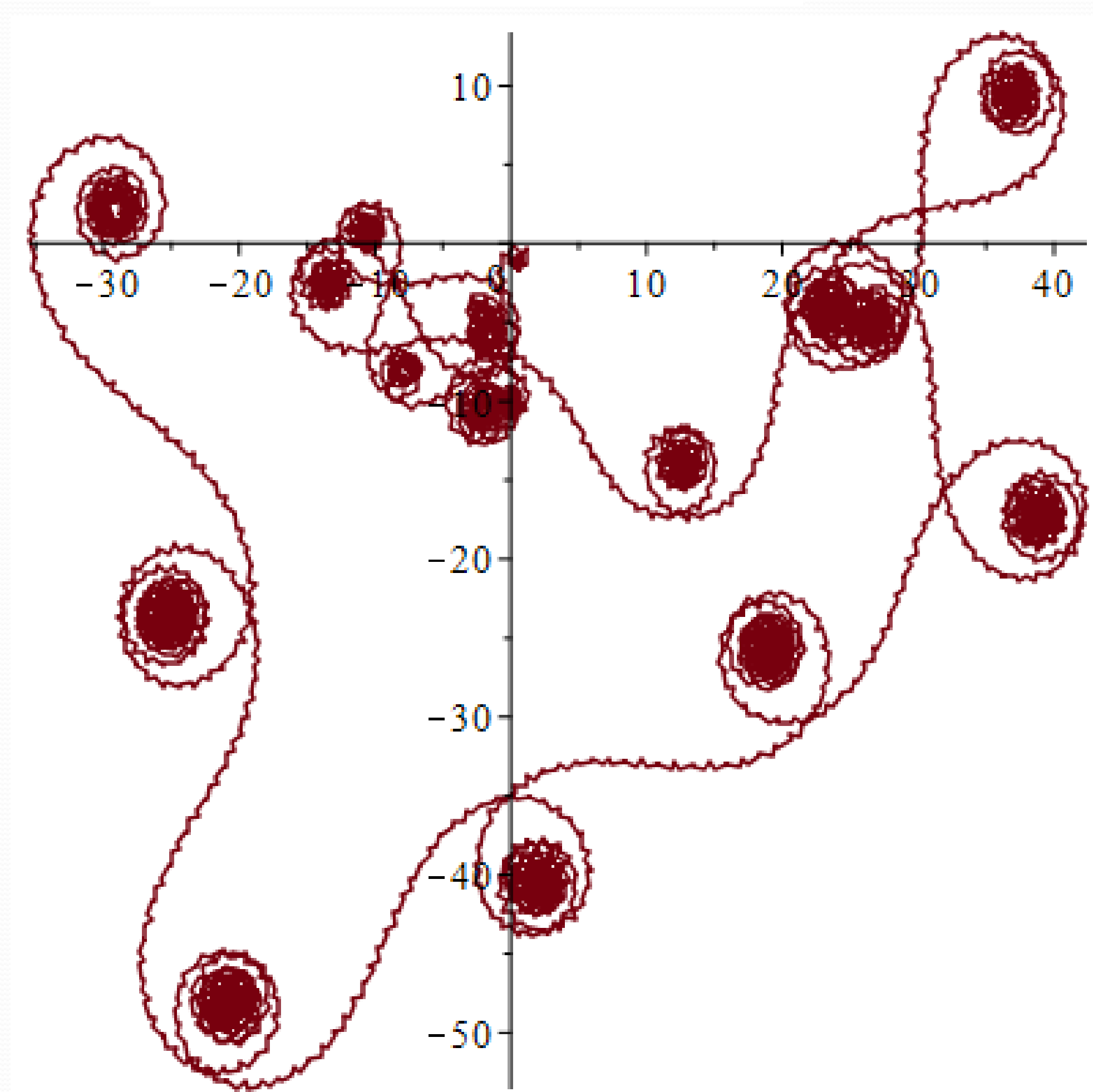
$n \mapsto \sum_{k=0}^n \omega^k \underbrace{\exp(2\pi i k / 29)}_{\text{PERIODIC}}$   
 $\omega = (1+3i)/\sqrt{10}$



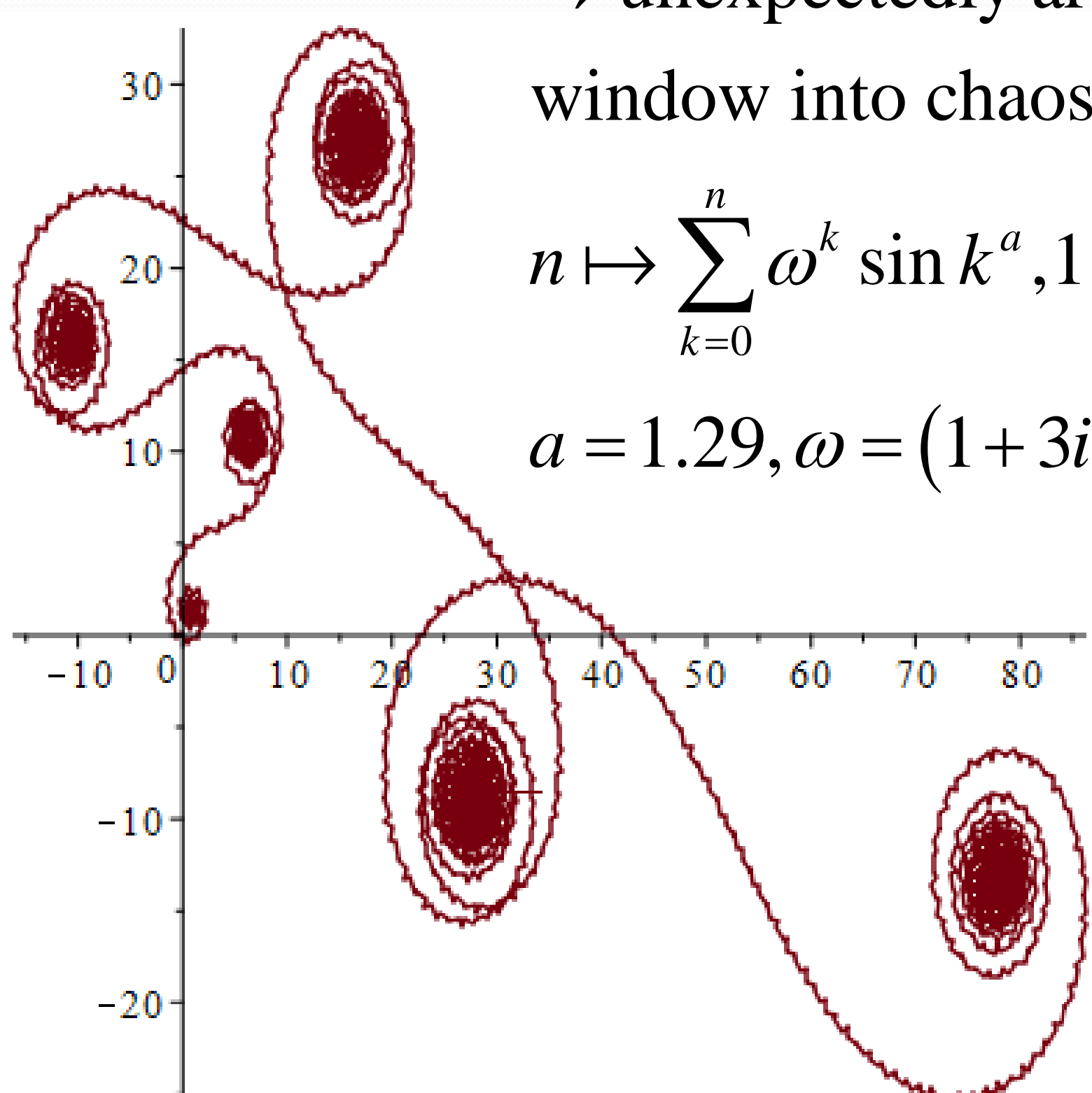
$n \mapsto \sum_{k=0}^n \omega^k \underbrace{\sin k}_{\text{NonPer}}$   
 $\omega = (1+3i)/\sqrt{10}$



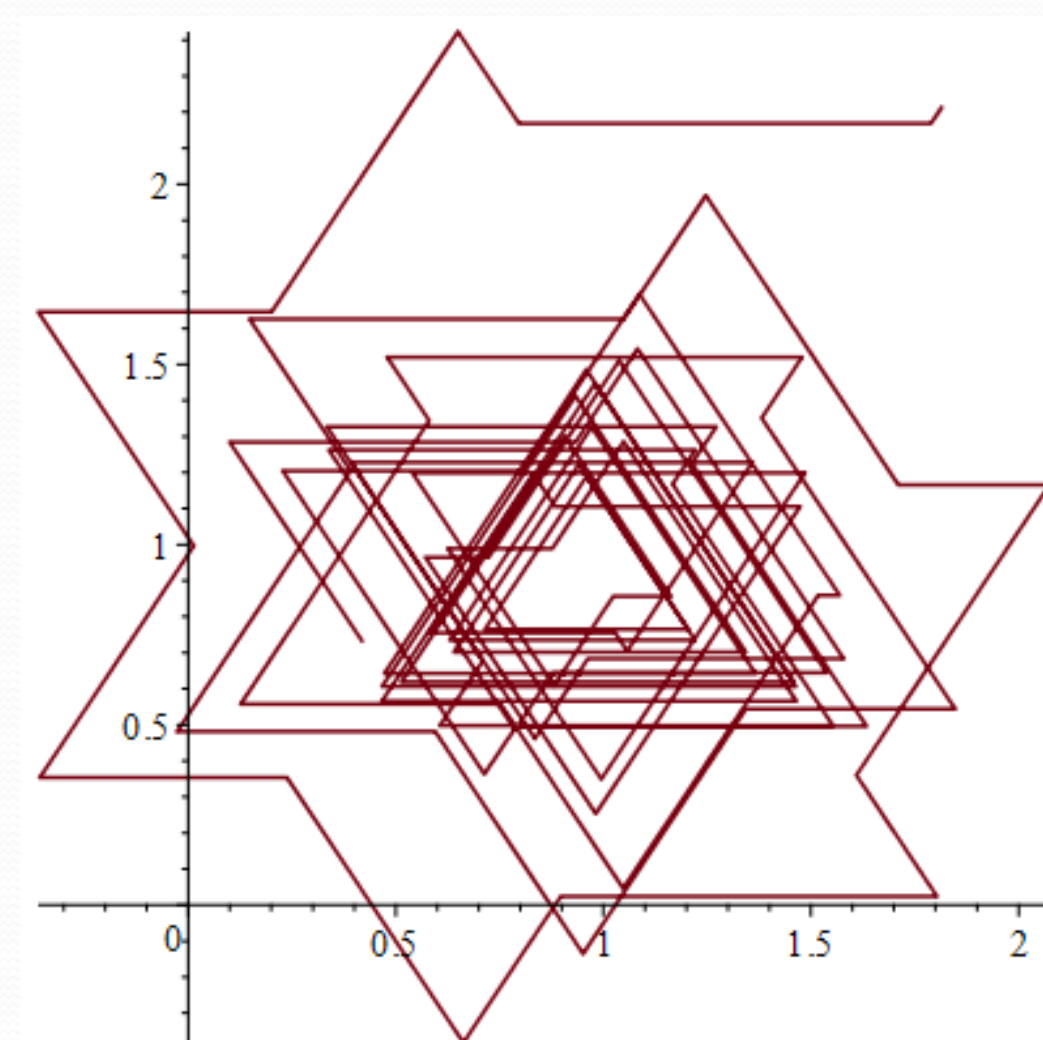
$n \mapsto \sum_{k=0}^n \omega^k \sin k^a$   
RAND  $\omega \cong -0.99 + 0.12i$   
RAND  $a \cong 1.856$   
not unexpected!



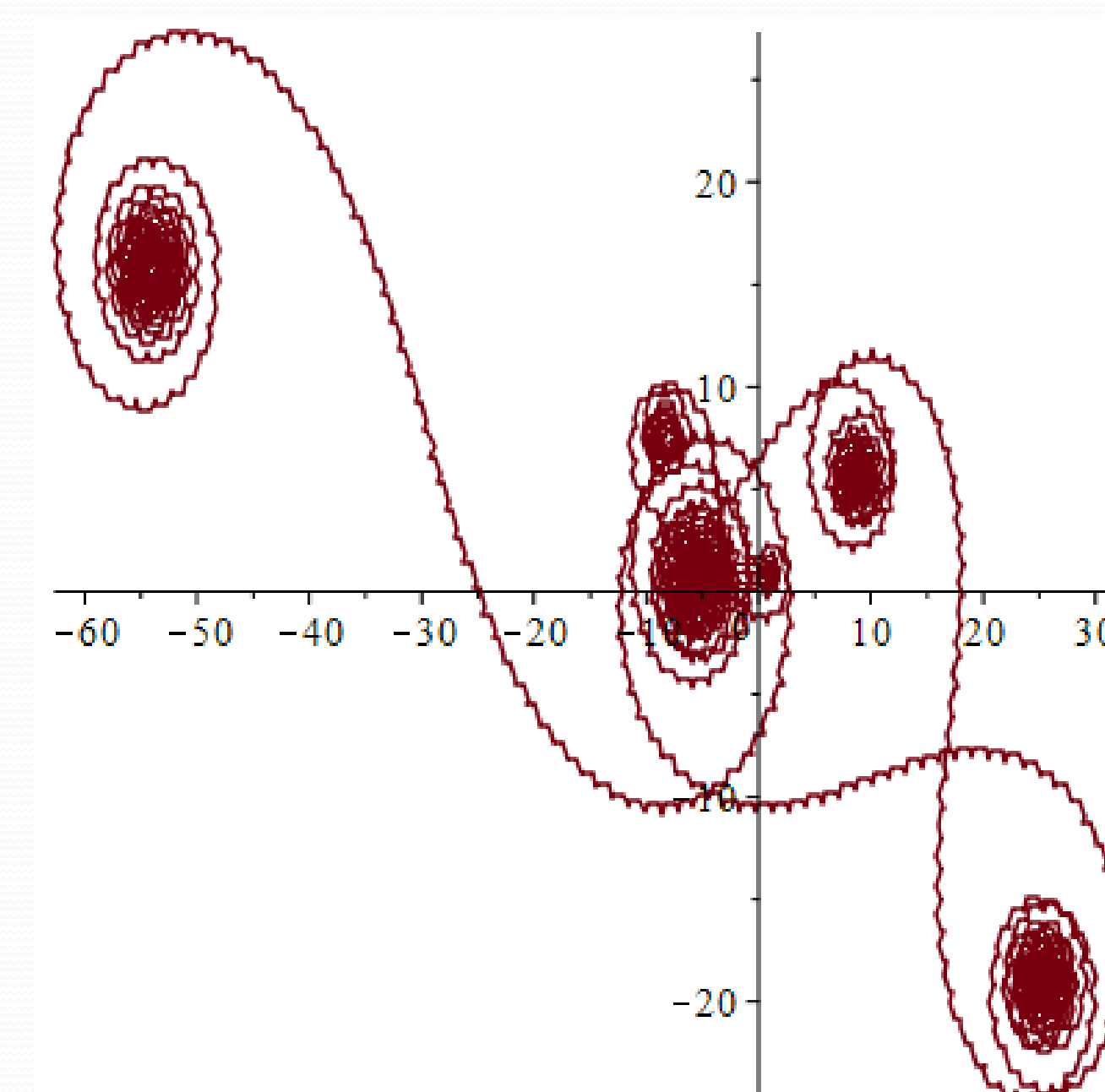
→ chaos emerges for other exponents!



→ unexpectedly artsty  
window into chaos!  
 $n \mapsto \sum_{k=0}^n \omega^k \sin k^a, 1 \leq n \leq 10000$   
 $a = 1.29, \omega = (1+3i)/\sqrt{10}$



$n \mapsto \sum_{k=0}^n \omega^k \sin k^a, 1 \leq n \leq 100$   
 $a = 1.29, \omega = \frac{1+i\sqrt{3}}{2} (\omega^6 = 1)$   
small range hexagonal SYM



SAME, but  $1 \leq n \leq 10000$   
LOCAL hex symmetry

$n \mapsto \sum_{k=0}^n \omega^k \sin k^a$   
 $1 \leq n \leq 10000$   
 $a = 1.4005$  RANDOM  
 $\omega = 0.2975 - 0.9547i$  RANDOM