Apr 22nd, 11:00 AM - 12:00 PM

Artsy chaos: the secret life of a class of trigonometric sums

Kaleb Swieringa
Ohio Northern University

Joelena Brown
Ohio Northern University

Rachael Harbaugh
Ohio Northern University

Francis Nadolny
Ohio Northern University

ARTSY CHAOS: The Secret Life of a Class of Trigonometric Sums

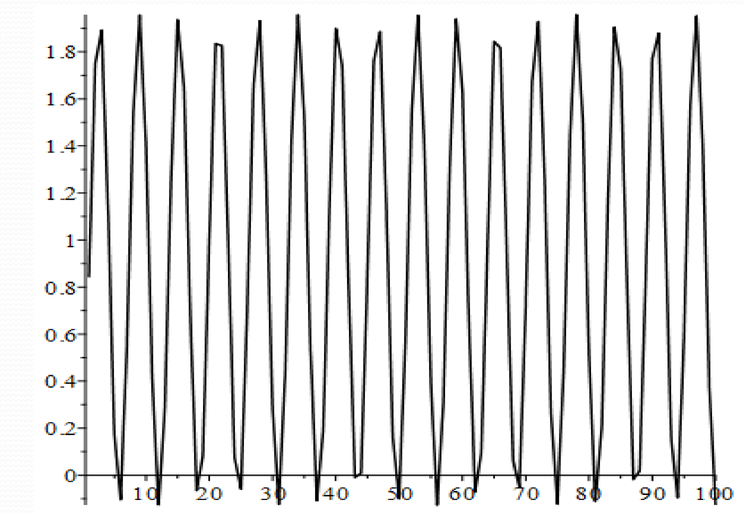
STUDENT RESEARCHERS: Kaleb Swieringa†, Joelena Brown, Rachael Harbaugh and Francis Nadolny
ADVISOR: Dr. Mihai Caragiu

†Corresponding author (k-swieringa@onu.edu)

ABSTRACT: *Classical trigonometric sums of the form* $\sum_{k=1}^{n}\cos k$ *and* $\sum_{k=1}^{n}\sin k$ *have simple closed form representations. Our work considered changing the arguments of the trigonometric factors from* $\cos k$ *(or* $\sin k$*) to nonlinear analogues* $\cos k^{\alpha}$ *(or* $\sin k^{\alpha}$*), modulated with rotational terms of the form* ω^k *for some complex* ω *of modulus 1. As an interesting outcome, for some exponents* α *(such as* $\alpha \cong 1.29$*, a case that we studied in more detail) we discover surprisingly esthetic/"artsy" chaotic complex plots/regimes (courtesy of MAPLE)*

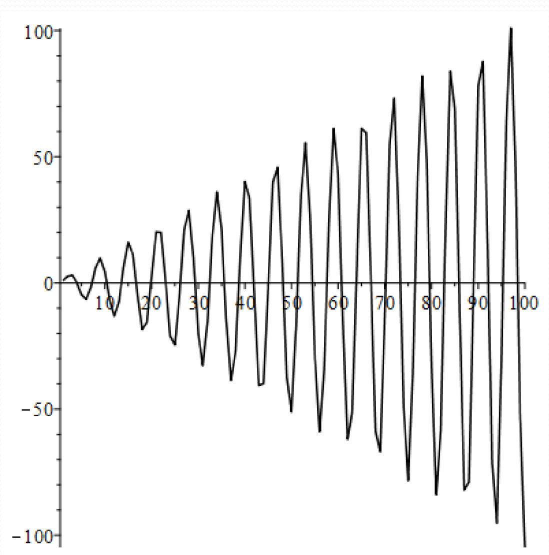
Classical trigonometric sums (straightforward, with $\exp(ix) = \cos x + i\sin x$):

$$C_n := \sum_{k=0}^{n}\cos k = \frac{\sin\left(\frac{n+1}{2}\right)\cos\left(\frac{n}{2}\right)}{\sin\left(\frac{1}{2}\right)} \quad \text{and} \quad S_n := \sum_{k=0}^{n}\sin k = \frac{\sin\left(\frac{n+1}{2}\right)\sin\left(\frac{n}{2}\right)}{\sin\left(\frac{1}{2}\right)}$$

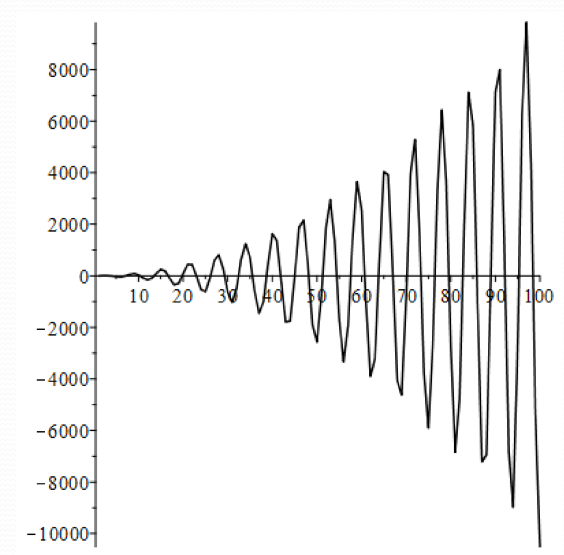


$$\textit{Ex. } S_n = \frac{1}{2}\cot\left(\frac{1}{2}\right) - \frac{\cos\left(n+\frac{1}{2}\right)}{2\sin\left(\frac{1}{2}\right)}$$

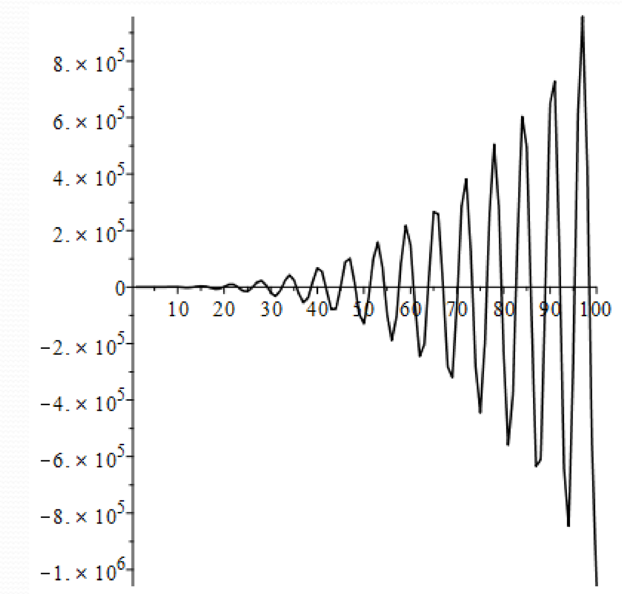
clear, controlled oscillatory range



$$n \mapsto \sum_{k=0}^{n} k\sin k$$

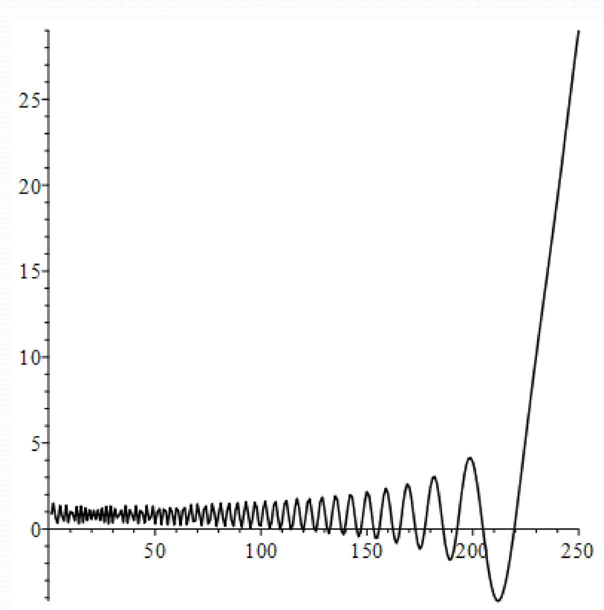


$$n \mapsto \sum_{k=0}^{n} k^2\sin k$$



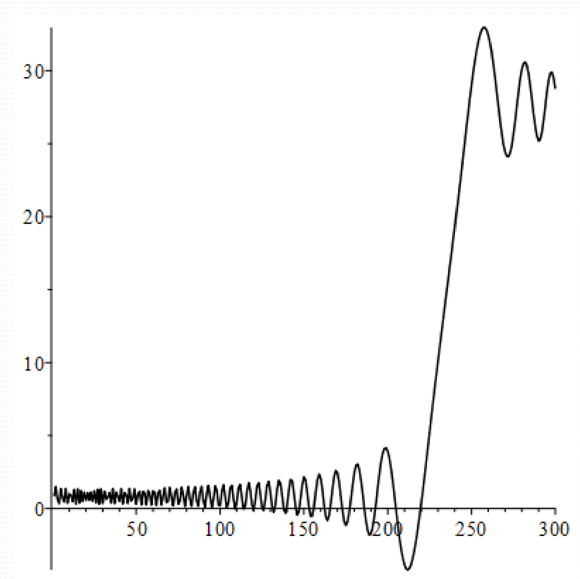
$$n \mapsto \sum_{k=0}^{n} k^3\sin k$$

→ again, deterministic, expected



$$n \mapsto \sum_{k=0}^{n}\sin\left(k^{1.29}\right)$$

$1 \le n \le 250$

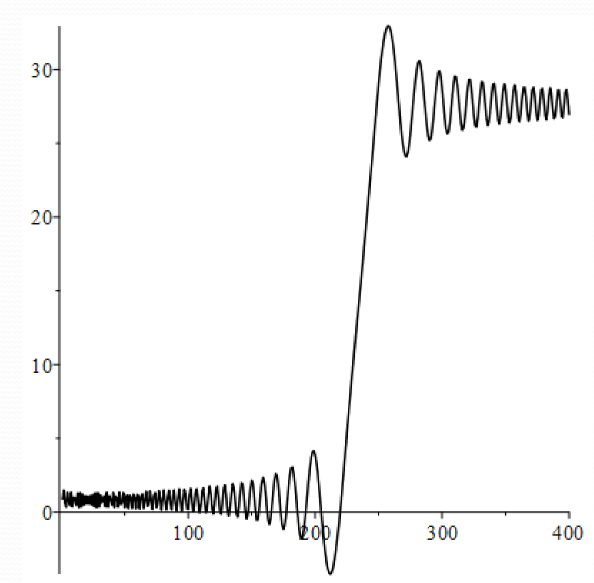


$$n \mapsto \sum_{k=0}^{n}\sin\left(k^{1.29}\right)$$

$1 \le n \le 300$

shifts oscillatory regime!

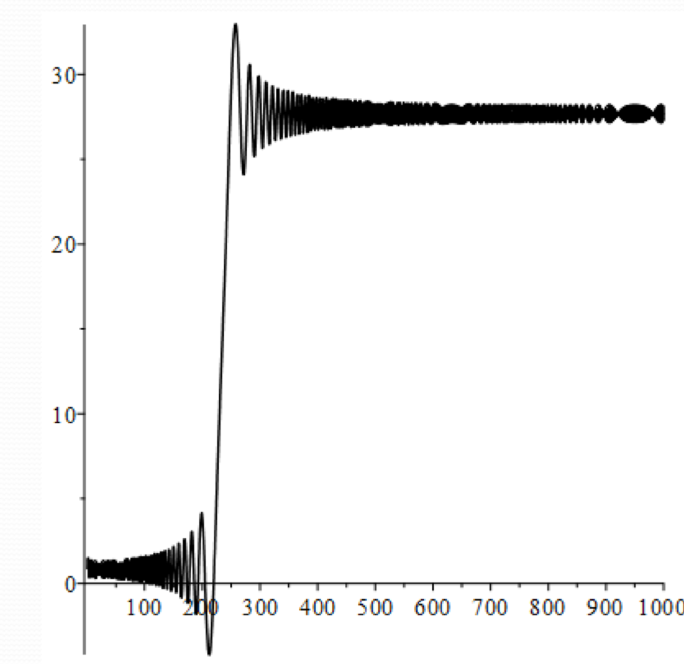
sort of unexpected

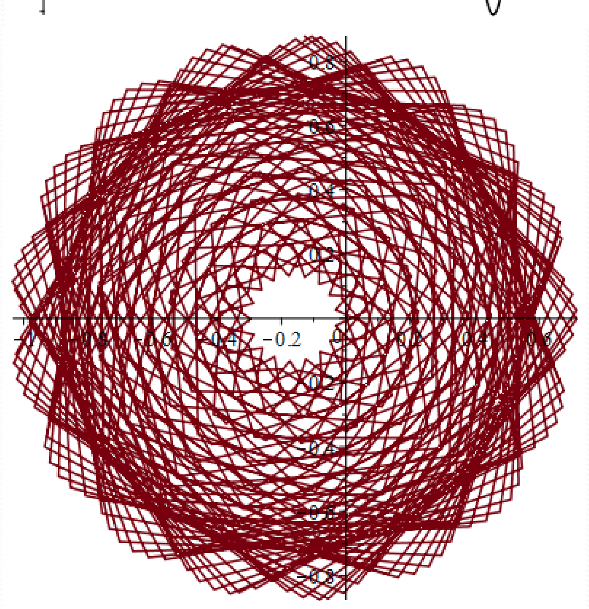


$$n \mapsto \sum_{k=0}^{n}\sin\left(k^{1.29}\right)$$

$1 \le n \le 400$

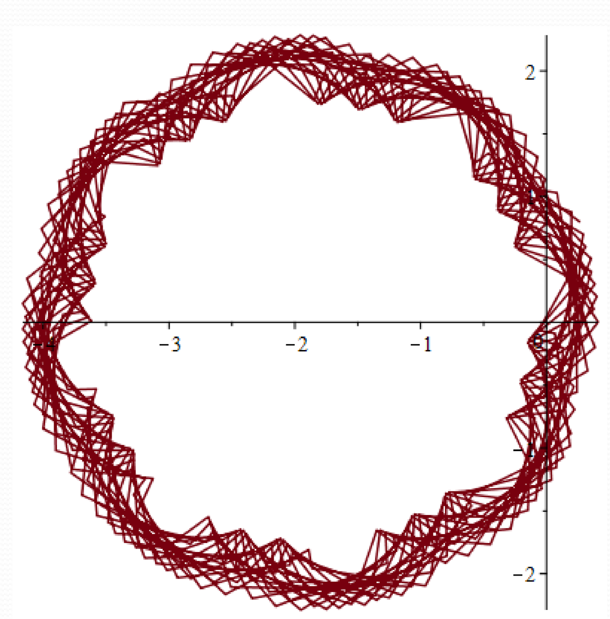
locally stable shift!





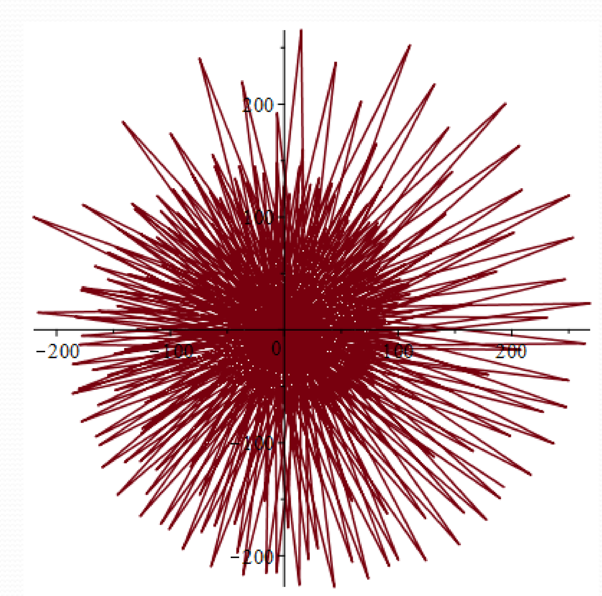
$$n \mapsto \sum_{k=0}^{n}\omega^k \underbrace{\exp(2\pi i k/29)}_{PERIODIC}$$

$\omega = (1+3i)/\sqrt{10}$



$$n \mapsto \sum_{k=0}^{n}\omega^k \underbrace{\sin k}_{NonPer}$$

$\omega = (1+3i)/\sqrt{10}$

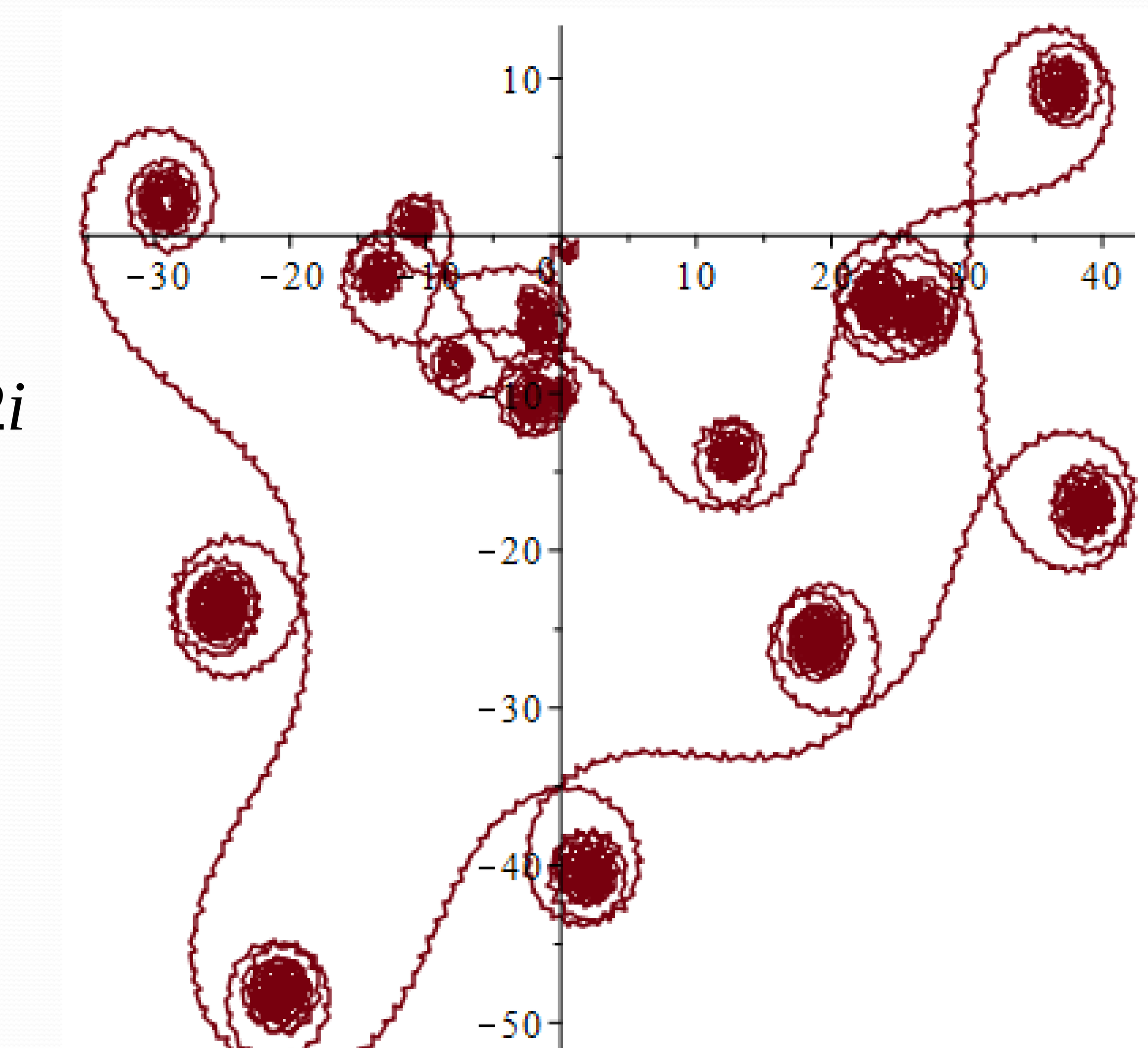


$$n \mapsto \sum_{k=0}^{n}\omega^k \sin k^a$$

RAND $\omega \cong -0.99 + 0.12i$

RAND $a \cong 1.856$

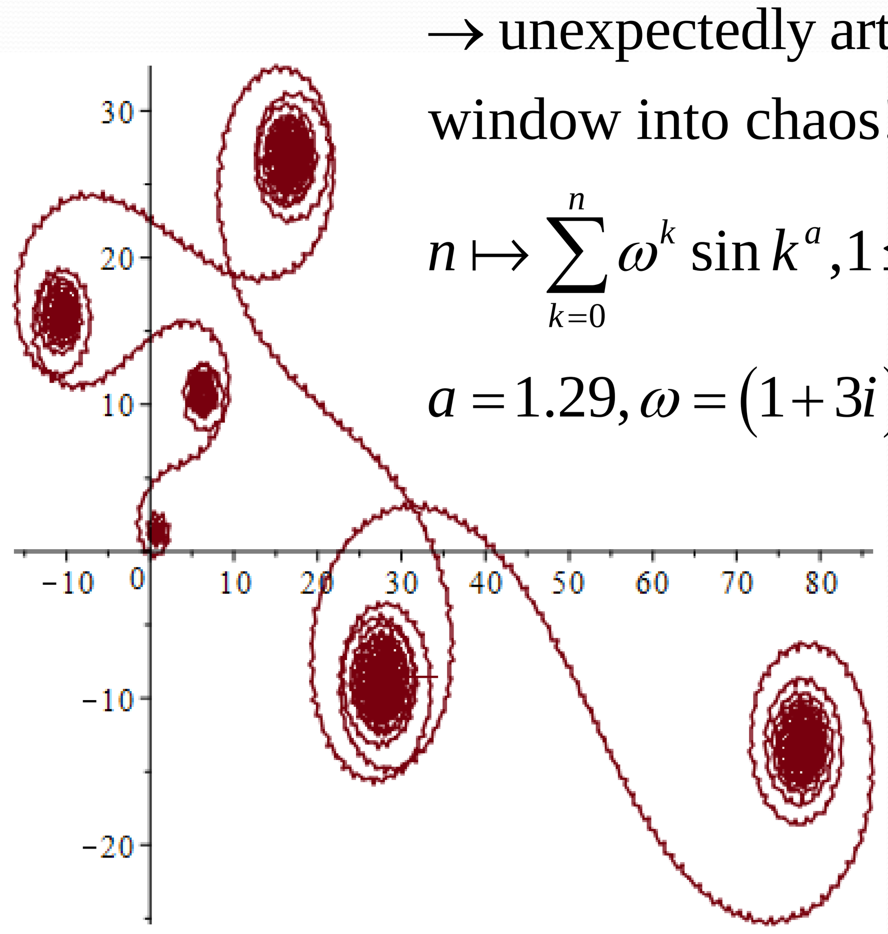
not unexpected!

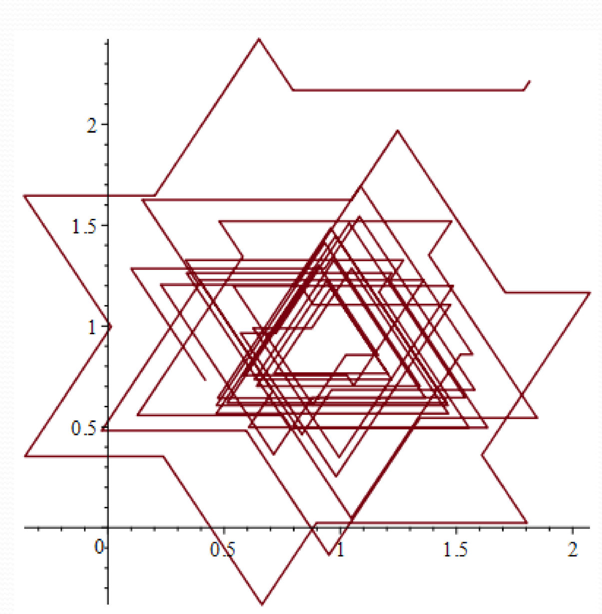


→ unexpectedly artsy window into chaos!

$$n \mapsto \sum_{k=0}^{n}\omega^k \sin k^a, 1 \le n \le 10000$$

$a = 1.29, \omega = (1+3i)/\sqrt{10}$

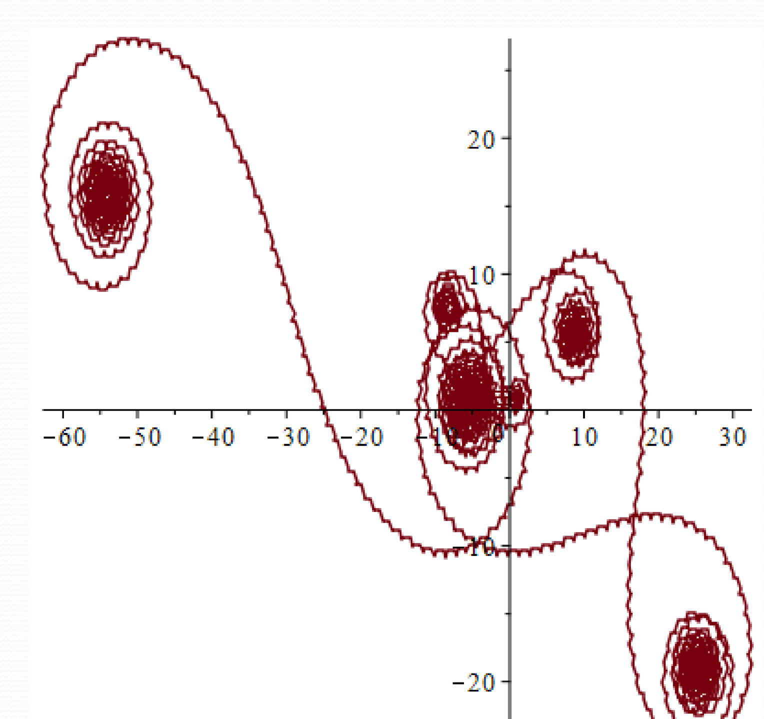




$$n \mapsto \sum_{k=0}^{n}\omega^k \sin k^a, 1 \le n \le 100$$

$$a = 1.29, \omega = \frac{1+i\sqrt{3}}{2}\ (\omega^6 = 1)$$

small range hexagonal SYM



SAME, but $1 \le n \le 10000$

LOCAL hex symmetry

→ chaos emerges for other exponents!

$$n \mapsto \left.\sum_{k=0}^{n}\omega^k \sin k^a\right|_{\omega = 0.2975 - 0.9547i \text{ RANDOM}}^{a = 1.4005 \text{ RANDOM}}$$

$1 \le n \le 10000$